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PUBLICATIONS OF THE AERO-ASTRONAUTICS GROUP 1965-76

by

E.C. WILSON



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RICE UNIVERSITY

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| | E.C. WILSON ² | |

Abstract. This document summarizes the research performed by the Aero-Astronautics Group of Rice University during the period 1965-76 under several AFOSR, NSF, and NASA grants. This research has been reported in 133 Aero-Astronautics Reports and 91 papers published in the open literature. It has spanned the following mathematical areas: (i) nonlinear equations, (ii) differential equations, (iii) two-point and multipoint boundary-value problems, (iv) mathematical programming, (v) optimal control, and (vi) calculus of variations. In these areas, it has led to the development of several new analytical and computational techniques.

Concerning applications, the research reported here is of interest in several areas of engineering, science, and economics. With particular regard to aerospace engineering, it applies to the following problem areas: (i) optimum atmospheric flight trajectories, (ii) optimum extra-atmospheric flight trajectories, (iii) optimum aerodynamic shapes, and (iv) optimum structures.

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² Secretary, Department of Mechanical Engineering and Materials Science, Rice University, Houston, Texas.

<u>Key Words.</u> Nonlinear equations, differential equations, two-point boundary-value problems, multipoint boundary-value problems, mathematical programming, optimal control, calculus of variations.

Numerical analysis, numerical methods, computing methods, computing techniques.

Systems theory, engineering systems, aerospace engineering, economics.

Optimum systems, optimum atmospheric flight trajectories, optimum extra-atmospheric flight trajectories, optimum aerodynamic shapes, optimum structures.

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I. Introduction

This document summarizes the research performed by the Aero-Astronautics Group of Rice University during the period 1965-76. This research has been supported through the following US Government Grants:

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NASA Grant No. NGR-44-006-063, 1967-68

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The personnel participating in the research effort included the following people:

Faculty Personnel

Prof. A. Miele

Prof. H.Y. Huang

Senior Personnel

Dr. A. Calabro
Dr. J.N. Damoulakis
Dr. A. Mangiavacchi
Dr. A. Montalvo
Dr. V. Guerra
Dr. R.E. Pritchard
Dr. J.C. Heideman
Dr. F. Rossi
Dr. R.R. Iyer
Major G.R. Hennig, USAF

Junior Personnel

| Mr. A.K. Aggarwal | Mr. D.G. Hull |
|---------------------|---------------------|
| Mr. F. Bonardo | Mr. R.R. Iyer |
| Mr. S.L. Brown | Mr. A.V. Levy |
| Mr. J.W. Cantrell | Mr. C.T. Liu |
| Mr. J.P. Chambliss | Mr. A.H. Lusty, Jr. |
| Mr. J.R. Cloutier | Mr. B.P. Mohanty |
| Mr. G.M. Coggins | Mr. P.E. Moseley |
| Mr. E.E. Cragg | Mr. S. Naqvi |
| Mr. J.N. Damoulakis | Mr. R. E. Pritchard |
| Mr. A. Esterle | Mr. J.L. Tietze |
| Mr. S. Gonzalez | Mr. K.H. Well |
| Mr. J.C. Heideman | Mr. W.L. Wilson |
| Mr. H.Y. Huang | Mr. A.K. Wu |

As a partial result of research performed under the above grants, the following advanced degrees were awarded:

MS Degrees

| A.K. Aggarwal | J.C. Heideman |
|----------------|---------------|
| J.W. Cantrell | H.Y. Huang |
| J.P. Chambliss | A.V. Levy |
| J.R. Cloutier | S. Naqvi |
| G.M. Coggins | W.L. Wilson |
| E.E. Cragg | A.K. Wu |
| | |

PhD Degrees

| A.K. Aggarwal | A.V. Levy |
|-----------------|-----------------|
| J.R. Cloutier | A.H. Lusty, Jr. |
| E.E. Cragg | P.E. Moseley |
| J.N. Damoulakis | S. Naqvi |
| J.C. Heideman | R.E. Pritchard |
| H.Y. Huang | J.L. Tietze |
| D.G. Hull | K.H. Well |
| R.R. Iyer | |
| | |

Over the period 1965-76, the research of the Aero-Astronautics Group has been concerned with the foliowing mathematical areas: (i) nonlinear equations, (ii) differential equations, (iii) two-point and multipoint boundary-value problems, (iv) mathematical programming, (v) optimal control, and (vi) calculus of variations. In these areas, it has led to the development of several new analytical and computational techniques.

Concerning applications, the research reported here is of interest in several areas of engineering, science, and economics. With particular regard to aerospace engineering, it applies to the following problem areas: (i) optimum atmospheric flight trajectories, (ii) optimum extra-atmospheric flight trajectories,

(iii) optimum aerodynamic shapes, and (iv) optimum structures.

A list of the research reports of the Aero-Astronautics Group is given in Section II. In turn, Section III contains a list of the papers published in the open literature by members of the Aero-Astronautics Group.

Remark. Aero-Astronautics Report (AAR) and Aero-Astronautics Papers (AAP) can be obtained by writing to the following address:

Dr. Angelo Miele Aero-Astronautics Group 230 Ryon Building Rice University Houston, Texas 77001

- II. Reports of the Aero-Astronautics Group
- AAR-1. MIELE, A., Extremal Problems in Aerodynamics, Rice University, Aero-Astronautics Report No. 1, 1965.
- AAR-2. MIELE, A., Generalized Approach to the Calculus of Variations in Two

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- AAR-3. HULL, D.G., and MIELE, A., Three-Dimensional Wings of Minimum

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- AAR-4. MIELE, A., Similarity Laws for Optimum Hypersonic Bodies, Rice University, Aero-Astronautics Report No. 4, 1965.
- AAR-5. HULL, D.G., Three-Dimensional Configurations of Minimum Total Drag
 in Newtonian Flow, Rice University, Aero-Astronautics Report No. 5,

 1965.
- AAR-6. HULL, D.G., and MIELE, A., Three-Dimensional Hypersonic Shapes
 of Minimum Total Drag, Rice University, Aero-Astronautics Report
 No. 6, 1965.
- AAR-7. MIELE, A., and PRITCHARD, R.E., Optimum Slender Bodies in Free-Molecular Flow, Rice University, Aero-Astronautics Report No. 7, 1965.
- AAR-8. MIELE, A., Optimum Transversal Contour of a Nonlifting Body in Newtonian Flow, Rice University, Aero-Astronautics Report No. 8, 1965.

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- AAR-9. MIELE, A., <u>Lift-to-Drag Ratios of Slender Bodies at Hypersonic Speeds</u>,
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- AAR-10. MIELE, A., and HULL, D.G., Maximum Lift-to-Drag Ratios of Slender,

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 Report No. 10, 1965.
- AAR-11. MIELE, A., Similarity Laws for Lifting Bodies of Minimum Drag at

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 1965.
- AAR-12. MIELE, A., Extremization of Products of Powers of Functionals, Rice University, Aero-Astronautics Report No. 12, 1966.
- AAR-13. MIELE, A., <u>Lift-to-Drag Ratios of Slender Wings at Hypersonic Speeds</u>,
 Rice University, Aero-Astronautics Report No. 13, 1966.
- AAR-14. MIELE, A., One-Dimensional Approach to the Maximum Lift-to-Drag

 Ratio of a Slender, Flat-Top, Hypersonic Wing, Rice University, AeroAstronautics Report No. 14, 1966.
- AAR-15. MIELE, A., <u>Two-Dimensional Approach to the Maximum Lift-to-Drag</u>

 <u>Ratio of a Slender, Flat-Top, Hypersonic Wing</u>, Rice University, AeroAstronautics Report No. 15, 1966.
- AAR-16. MIELE, A., Similarity Laws for Lifting Wings of Minimum Drag at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 16, 1966.
- AAR-17. MIELE, A., Maximum Lift-to-Drag Ratio of a Nonslender, Flat-Top,

 Hypersonic Wing, Rice University, Aero-Astronautics Report No. 17, 1966.

- AAR-18. MIELE, A., and HULL, D.G., Sufficiency Proofs for the Problem of the Optimum Transversal Contour, Rice University, Aero-Astronautics Report No. 18, 1966.
- AAR-19. MIELE, A., Simplified Approach to the Problem of the Optimum Transversal Contour, Rice University, Aero-Astronautics Report No. 19, 1966.
- AAR-20. MIELE, A., and HUANG, H.Y., Power-Law Bodies of Maximum Liftto-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 20, 1966.
- AAR-21. LUSTY, A.H., Jr., Lifting Bodies of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 21, 1966.
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 Report No. 26, 1966.
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- AAR-35. HUANG, H.Y., Variational Approach to Conical Bodies Having Maximum

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- AAR-54. MIELE, A., Summary Report on General Study of Optimum Aerodynamic

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- AAR-56. MIELE, A., and CANTRELL, J.W., Gradient Methods in Mathematical

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- AAR-57. DAMOULAKIS, J.N., The Restoration of Constraints in Nonholonomic

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- AAR-58. CRAGG, E.E., and LEVY, A.V., Gradient Methods in Mathematical

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- AAR-64. HUANG, H.Y., Unified Approach to Quadratically Convergent Algorithms

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- AAR-65. DAMOULAKIS, J.N., Gradient Methods in Control Theory, Part 3, Sequential Gradient-Restoration Algorithm: Numerical Examples, Rice University, Aero-Astronautics Report No. 65, 1969.
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- AAR-68. MIELE, A., HEIDEMAN, J.C., and LEVY, A.V., Mathematical Programming for Constrained Minimal Problems, Part 3, Combined Gradient-Restoration Algorithm, Rice University, Aero-Astronautics Report No. 68, 1970.
- AAR-69. HEIDEMAN, J.C., and LEVY, A.V., Mathematical Programming for

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- AAR-70. MIELE, A., HEIDEMAN, J.C., and LEVY, A.V., Mathematical Programming for Constrained Minimal Problems, Part 5, Combined Conjugate Gradient-Restoration Algorithm, Rice University, Aero-Astronautics Report No. 70, 1970.

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 Report No. 71, 1970.
- AAR-72. MIELE, A., LEVY, A.V., and CRAGG, E.E., Mathematical Programming for Constrained Minimal Problems, Part 6, Modifications and Extensions of the Conjugate Gradient-Restoration Algorithm, Rice University, Aero-Astronautics Report No. 72, 1970.
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- AAR-74. MIELE, A., Gradient Methods in Control Theory, Part 6, Combined

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- AAR-75. MIELE, A., CRAGG, E.E., IYER, R.R., and LEVY, A.V., Use of the

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- AAR-77. MIELE, A., IYER, R.R., and WELL, K.H., Modified Quasilinearization and Optimal Initial Choice of the Multipliers, Part 2, Optimal Control Problems, Rice University, Aero-Astronautics Report No. 77, 1970.

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- AAR-82. HUANG, H.Y., and NAQVI, S., Unconstrained Approach to the Extremization of Constrained Functions, Rice University, Aero-Astronautics Report No. 82, 1970.
- AAR-83. WELL, K.H., Note on a Problem by Lance and a Problem by Bellman, Rice University, Aero-Astronautics Report No. 83, 1971.
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- AAR-87. HUANG, H.Y., and CHAMBLISS, J.P., Quadratically Convergent Algorithms and One-Dimensional Search Schemes, Rice University, Aero-Astronautics Report No. 87, 1972.
- AAR-88. HUANG, H.Y., Method of Dual Matrices for Function Minimization, Rice University, Aero-Astronautics Report No. 88, 1972.
- AAR-89. HUANG, H.Y., and CHAMBLISS, J.P., Numerical Experiments on Dual

 Matrix Algorithms for Function Minimization, Rice University, AeroAstronautics Report No. 89, 1972.
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 Penalty Constant Used in the Penalty Function Method for Mathematical

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 No. 90, 1972.
- AAR-91. MIELE, A., Combined Gradient-Restoration Algorithm for Optimal Control Problems, Rice University, Aero-Astronautics Report No. 91, 1971.
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- AAR-99. MIELE, A., MOSELEY, P.E., LEVY, A.V., COGGINS, G.M., On the Method of Multipliers for Mathematical Programming Problems,

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- AAR-101. HENNIG, G.R., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with Bounded State Variables,

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19. KEY WORDS (Continued)

calculus of variations.

Numerical analysis, numerical methods, computing methods, computing techniques.

Systems theory, engineering systems, aerospace engineering, economics.

Optimum systems, optimum atmospheric flight trajectories, optimum extra-atmospheric flight trajectories, optimum aerodynamic shapes, optimum structures.

20. ABSTRACT (continued)

linear equations; (ii) differential equations; (iii) two-point and multipoint boundary-value problems; (iv) mathematical programming; (v) optimal control; and (vi) calculus of variations. In these areas, it has led to the development of several new analytical and computational techniques.

Concerning applications, the research reported here is of interest in several' areas of engineering, science, and economics. With particular regard to aerospace engineering, it applies to the following problem areas: (i) optimum atmospheric flight trajectories; (ii) optimum extra-atmospheric flight trajectories; (iii) optimum aerodynamic shapes; and (iv) optimum structures.

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